MATH 16A MIDTERM 1(PRACTICE 3)

PROFESSOR PAULIN

DO NOT TURN OVER UNTIL INSTRUCTED TO DO SO.
CALCULATORS ARE NOT PERMITTED
YOU MAY USE YOUR OWN BLANK PAPER FOR ROUGH WORK
SO AS NOT TO DISTURB OTHER STUDENTS, EVERYONE MUST STAY UNTIL THE EXAM IS COMPLETE
REMEMBER THIS EXAM IS GRADED BY A HUMAN BEING. WRITE YOUR SOLUTIONS NEATLY AND COHERENTLY, OR THEY RISK NOT RECEIVING FULL CREDIT
THIS EXAM WILL BE ELECTRONICALLY SCANNED. MAKE SURE YOU WRITE ALL SOLUTIONS IN THE SPACES PROVIDED. YOU MAY WRITE SOLUTIONS ON THE BLANK PAGE AT THE BACK BUT BE SURE TO CLEARLY LABEL THEM

Name and section: ______________________________

GSI's name: ______________________________

This exam consists of 5 questions. Answer the questions in the spaces provided.

1. Describe in words, how, starting with the graph $y = \sqrt{x}$, one can draw the graph

$$y = -\sqrt{2-x} + 1$$

Solution:

1/ $y = \sqrt{x} \longrightarrow y = \sqrt{2+x}$

Translate to left by 2

2/ $y = \sqrt{2+x} \longrightarrow y = \sqrt{2-x}$

Reflect in y-axis

3/ $y = \sqrt{2-x} \longrightarrow y = -\sqrt{2-x}$

Reflect in x-axis

4/ $y = -\sqrt{2-x} \longrightarrow y = -\sqrt{2-x} + 1$

Translate up by 1

2. (25 points) A bank has a saving account in why interest is compounded continuously. If the amount in the account doubles every 30 years determine what the annual interest rate is. How long will it take for the balance to triple? You do not need to simplify your answers.

Solution:

P = initial deposit

r = annual interest rate

$f(t) = Pe^{rt}$ = balance at time t.

$$f(30) = 2f(0) \Rightarrow Pe^{30r} = 2P$$

$$\Rightarrow e^{30r} = 2 \Rightarrow r = \frac{\ln(2)}{30}.$$

$$f(t) = 3P \Rightarrow Pe^{\frac{\ln(2)}{30}t} = 3P$$

$$\Rightarrow t = \frac{\ln(3)}{\left(\frac{\ln(2)}{30}\right)} = \frac{30\ln(3)}{\ln(2)}$$

3. Calculate the following limits. If they do not exist determine if they are ∞ or $-\infty$.

(a)

$$\lim_{x\to 1} 2^{x^2+x^3+1}$$

Solution:

$$\lim_{x\to 1} x^2+x^3+1 = 1^2+1^3+1 = 3$$

$$\Rightarrow \lim_{x\to 1} 2^{x^2+x^3+1} = 2^3$$

(b)

$$\lim_{x\to -\infty} \frac{6x^3+1}{x+3}$$

Solution:

$$\lim_{x\to -\infty} \frac{6x^3+1}{x+3} = \lim_{x\to -\infty} \frac{6x^3}{x} = \lim_{x\to -\infty} 6x^2 = \infty$$

(DNE)

(c)

$$\lim_{x\to 2} \frac{\ln(x^2+1)}{(x-2)^3}$$

Solution:

$$\lim_{x\to 2} \ln(x^2+1) = \ln\left(\lim_{x\to 2}(x^2+1)\right) = \ln(5) > 0$$

$$\lim_{x\to 2} (x-2)^3 = 0$$

2

$(x-2)^3 < 0$ $\quad$ $(x-2)^3 > 0$

$$\Rightarrow \lim_{x\to 2^-} (x-2)^3 = 0^-$$

$$\lim_{x\to 2^+} (x-2)^3 = 0^+$$

$$\Rightarrow \lim_{x\to 2^+} \frac{\ln(x^2+1)}{(x-2)^3} = \infty$$

$$\lim_{x\to 2^-} \frac{\ln(x^2+1)}{(x-2)^3} = -\infty$$

$$\Rightarrow \lim_{x\to 2} \frac{\ln(x^2+1)}{(x-2)^3}$$ DNE and is neither $\pm\infty$.

4. (25 points) Using limits, calculate the derivative of $f(x) = \frac{(x-1)^{1/2}}{2}$ for $x > 1$. Using this, or otherwise, determine the equation of the tangent line to $y = f(x)$ at $x = 2$.

Solution:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\sqrt{x+h-1} - \sqrt{x-1}}{2h}$$

$$= \lim_{h \to 0} \frac{\left(\sqrt{x+h-1} - \sqrt{x-1}\right)\left(\sqrt{x+h-1} + \sqrt{x-1}\right)}{2h\left(\sqrt{x+h-1} + \sqrt{x-1}\right)}$$

$$= \lim_{h \to 0} \frac{(x+h-1) - (x-1)}{2h\left(\sqrt{x+h-1} + \sqrt{x-1}\right)}$$

$$= \lim_{h \to 0} \frac{1}{2\left(\sqrt{x+h-1} + \sqrt{x-1}\right)}$$

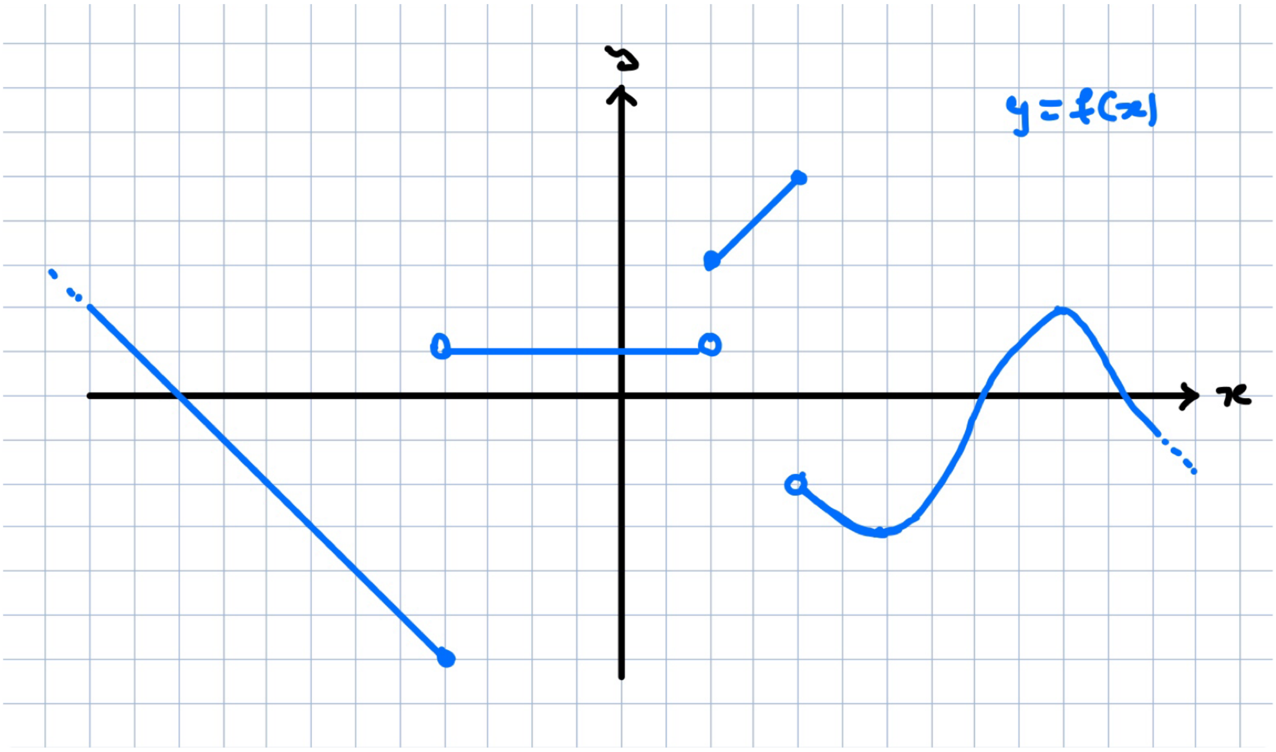
$$= \frac{1}{2\left(\sqrt{x-1} + \sqrt{x-1}\right)} = \frac{1}{4\sqrt{x-1}}$$

$$f'(2) = \frac{1}{4}, \quad f(2) = \frac{1}{2}$$

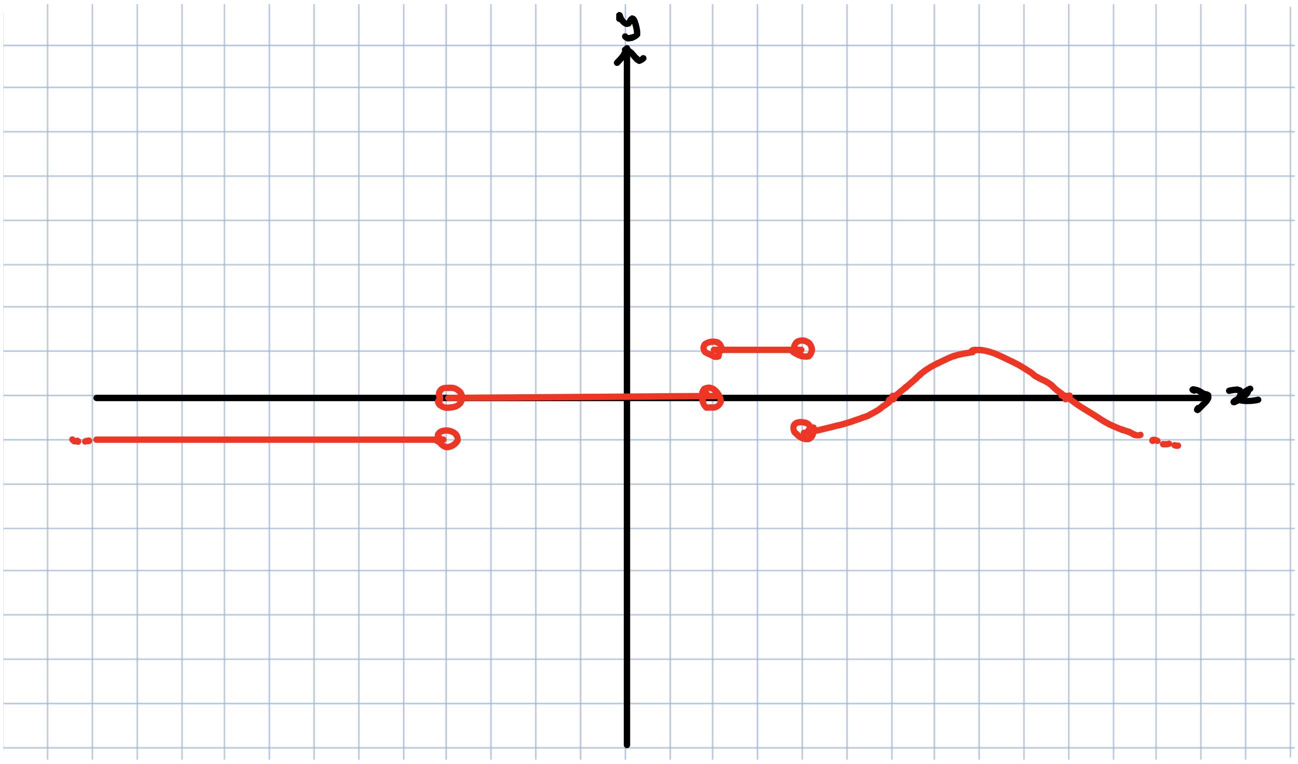
$\Rightarrow$ Tangent at $x = 2$ has equation

$$y - \frac{1}{2} = \frac{1}{4}(x-2)$$

5. Using the method of graphical differentiation roughly plot the graph of the derivative of the following function:



Solution:



END OF EXAM